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PAPER

Quantum contextuality of a single neutrino under interactions with matter

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Abstract

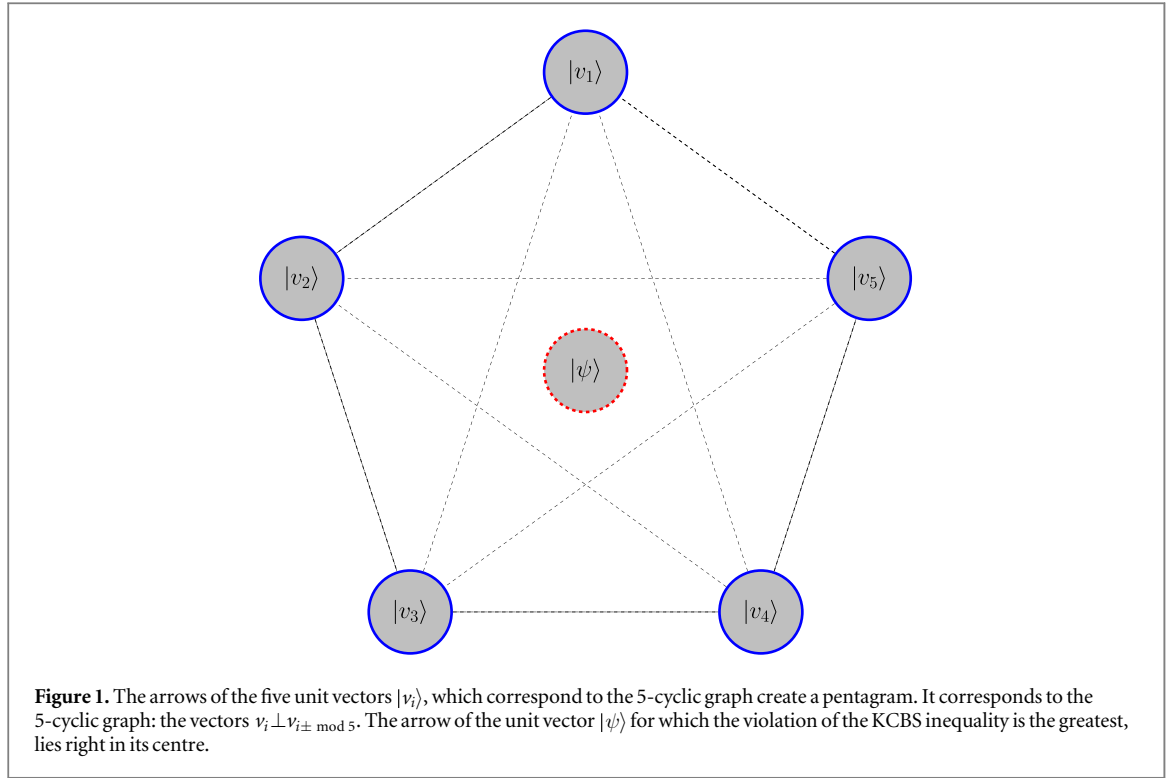
We study contextuality of the three neutrinos oscillations using the Klyachko–Can–Binicioglu–Shumovsky 5-observable test. We show how the interaction between neutrinos and normal matter affects contextuality—its loss and its possible revivals. We show that for open neutrino’s system, interacting with an environment, revivals of contextuality survive in a presence of decoherence included in the neutrino’s Lindblad–Kossakowski master equation in a simplest Markovian approximation.

1. Introduction

The most influential for quantum foundations triad—Einstein, Podolsky and Rosen [1]—claims that the physical theory should obey: (i) *locality*: the result of measurement carried out on the system **A** should not depend on the measurement performed on the spatially separated system **B** and (ii) *objective realism*: the properties of a system are set down before an act of measurement, which only reveals some pre-existing value. These requirements, if simultaneously satisfied, result in the celebrated EPR paradox. Quantum theory, objectively real and non-local, is recognized to be also *contextual*.

In our work we are interested in *contextuality* which, together with a temporal non-locality related to violation of the Leggett–Garg inequalities [2] and with the celebrated entanglement [3], is one of three strictly related [4, 5] ‘different types’ of non-locality. Contextuality was first introduced (without using the term ‘contextuality’) by Bell in [6] and Kochen and Specker in [7]. It is said that theory is *non-contextual* if for a given set A of physical quantities: $A = \{A_1, A_2, \dots, A_N\}$ each yielding respective measurements’ outcomes a_i , $i = 1, \dots, N$, the obtained values a_i does not depend on the measurements’ *context*, i.e. the set of another members of A measured along the quantity A_i [8]. In other words, there exists a joint probability distribution $p_{\text{joint}} = p(A_1 = a_1, A_2 = a_2, \dots, A_n = a_n)$ for the measurement of observables $\{A_1, A_2, \dots, A_n\}$ yielding the results $\{a_1, a_2, \dots, a_n\}$ and the probabilities of individual events can be calculated as the marginals of p_{joint} [8, 9].

An experimentally verifiable test for entanglement proposed by Bell [10] implemented in experiments performed by Alain Aspect and his collaborators [11] confirmed validity of quantum mechanics even though some physicists discerned the so-called ‘loopholes’ [12] opening the ‘door’ to the abolishment of quantum mechanics in favour of the hidden variable theories. These were taken into account in the future experiments [13, 14] which still, despite of some earlier doubts confirmed the Bell non-locality [9] and possibility of entanglement [3]. In the recent years many advances were also made towards experimental verification of the *quantum contextuality*. The experiments were carried out in the systems of trapped ions [15], photons [16], fermions [17], deuterium nucleus [18] or neutron [19]. Moreover, the quantum contextuality gains in the popularity also in the quantum computation’s community [20–22]. All these aspects indicate that the contextuality has become the active area of research. Roughly speaking the quantum contextuality tests verify if the expectation values of the set of observables can be described with the joint probability distribution [8]. There exist two classes of tests of the contextuality: state-dependent ones which provide an evidence for the non-



contextuality only for certain, often narrow, class of quantum states [23, 24] and state-independent test, which are valid for an arbitrary quantum state [25].

Due to its simplicity, in our work we focus only on the Klyachko–Can–Binicioglu–Shumovsky test (KCBS test) [24]. It is state-dependent [8] but requires only five observables to be measured and it is designed for the three-dimensional system [26] which is exactly the one considered in this paper. The KCBS inequality is constructed as follows. Let us suppose that our system is prepared in the pure state $|\psi\rangle$. One defines 5 projective measurements $\Pi_i = |v_i\rangle\langle v_i|$, $i = 1, \dots, 5$ such that the consecutive operators Π_i and $\Pi_{i \pm 1 \text{ mod } 5}$ are compatible, i.e. $[\Pi_i, \Pi_{i \pm 1 \text{ mod } 5}] = 0$. Then the *non-contextuality* condition for the system in the state $|\psi\rangle$ takes the following form [27]:

$$\sum_{i=1}^5 |\langle v_i | \psi \rangle|^2 \leq 2. \quad (1)$$

The vectors $|v_i\rangle$ form a regular pentagon, as presented in figure 1 [8, 24]. Quantum world however, although essentially contextual, does not always obey this inequality. For the qutrits [8] the maximum attainable value is $\max \sum_{i=1}^5 |\langle v_i | \psi \rangle|^2 = \sqrt{5} > 2$. It corresponds to the case when a state $|\psi\rangle$ lies in the centre of the pentagon created by the vectors $|v_i\rangle$ (see figure 1). An equivalent form of the KCBS inequality equation (1) is given in terms of a set of dichotomic (with eigenvalues $\lambda = \pm 1$) observables $A_i = 1 - 2|v_i\rangle\langle v_i|$ and reads:

$$\Delta = \langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle \geq -3. \quad (2)$$

The maximum violation of (2) admissible by the quantum mechanics occurs for $\Delta = -3.94$. An example of the vectors $|v_i\rangle$ and state $|\psi\rangle$ for which the violation of inequality (1) or equivalently (2) is extremely large is given below [8]:

$$\begin{aligned} |v_1\rangle &= N(\sqrt{c}, 1, 0) \\ |v_{2,5}\rangle &= N(\sqrt{c}, -c, \pm s) \\ |v_{3,4}\rangle &= N(\sqrt{c}, C, \mp S) \\ |\psi\rangle &= (1, 0, 0), \end{aligned} \quad (3)$$

charge conjugation parity (CP) where $N = 1/(\sqrt{1+c})$ stands for normalization constant, $c = \cos \alpha$, $s = \sin \alpha$, $C = \cos \beta$, $S = \sin \beta$, for the angles $\alpha = \pi/5$ and $\beta = 2\pi/5$.

Quantum non-locality of the Bell type related to quantum entanglement [28, 29] and the one of the Leggett–Garg type [30–32] have been studied recently in a context of oscillating neutrino systems. In particular, [33] reports violation of the LGI in neutrino systems based upon the Main Injector Neutrino Oscillation Search experiment’s data. In our theoretical work, we want to study a complementary topic and examine how the system of 3 oscillating neutrinos exhibit the contextuality in the presence of matter. Furthermore, we want to

investigate how the effect of this interaction becomes influenced by decoherence [34]. We show, although it may seem counter-intuitive, that the interaction with a normal matter modifying neutrino oscillations results in effective *revivals* of the quantum contextuality. Bearing in mind a high sophistication and extreme difficulties present in any neutrino measurement [33], we would like to emphasize purely qualitative character of our predictions. We modestly attempt to present the calculations which, as we believe, indicate general properties of the oscillating neutrino systems.

The paper is organized as follows: in the next section we review the qutrit model of neutrino's oscillation and apply it to investigate revivals of contextuality in section 3 originating from the interaction with matter. In section 4 we investigate an influence of decoherence and the contextuality revival in decohering environment. Finally we conclude our work.

2. Three-flavour neutrino oscillations

Neutrino oscillation is a phenomenon in which the neutrino of a given flavour $\alpha = \{e, \mu, \tau\}$, produced at some point can be further, after covering some distance, measured in a different flavour state. It is the direct consequence of the lack of 1–1 correspondence between flavour $\{\nu_e, \nu_\mu, \nu_\tau\}$ and massive neutrino states $\{\nu_1, \nu_2, \nu_3\}$. These two are connected via Pontecorvo–Maki–Nakagawa–Sakata unitary lepton mixing matrix U_{PMNS} :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (4)$$

where the matrix U_{PMNS} in the case of Dirac neutrino (considered in this paper) is parametrized with three mixing angles θ_{12} , θ_{23} , θ_{13} and one CP-violating phase δ [35]:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (5)$$

with $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$ (in calculations, following [36] we set $s_{12}^2 = 0.307$, $s_{13}^2 = 0.021$ and $s_{23}^2 = 0.5$) and δ being the CP-violating phase. As currently attainable sensitivity of experiments to estimate a value of δ in the following part of our work it is neglected and set to $\delta = 0$.

The time evolution of the neutrino is governed by the Schrödinger equation:

$$i \frac{d}{dt} \Psi_F = \mathcal{H}_F \Psi_F, \quad (6)$$

where Ψ_F describes the flavour neutrino state for a given time t and \mathcal{H}_F stands for the Hamiltonian [35, 37, 38]:

$$\mathcal{H}_F = H_{\text{kin}} + H_{\text{pot}}, \quad (7)$$

which naturally consists of the kinetic H_{kin} and H_{pot} potential parts to be described below.

In the flavour basis—taking into account an ultrarelativistic limit for the neutrinos (with neutrino's eigenenergies $E_k = E + \frac{m_k^2}{2E}$ and momentum $p \sim E$)—the kinetic Hamiltonian reads:

$$H_{\text{kin}} = \frac{1}{2E} U_{\text{PMNS}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U_{\text{PMNS}}^\dagger, \quad (8)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ denotes the difference between the masses of two oscillating neutrinos. In the following numerical calculations we set $\Delta m_{21}^2 = 7.37 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 = 2.52 \times 10^{-3} \text{ eV}^2$ [36].

As for the potential part, the main contribution to that comes from neutrino's interaction via the coherent forward elastic scattering (non-coherent effects are negligible) with the matter's electrons generating *charged-current potential* V_{CC} . There exists also interaction with the neutrons, but since it is the same for all neutrino's flavours, it does not affect the oscillations. Bearing all these facts in mind, the potential part takes the following form:

$$H_{\text{pot}} = \begin{pmatrix} V_{\text{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

Here the charged-current potential V_{CC} is related to the electron's density in the matter $V_{\text{CC}} = \sqrt{2} G_F n_e$ with G_F denoting the Fermi coupling constant and the electron number density n_e .

3. Contextuality of neutrinos treated as a closed system

Neutrinos are the simple example of the three-dimensional system which may exhibit *contextuality* i.e. their time evolution can lead to violation of the KCBS test equation (2) for a given set of measurements. It is therefore natural to ask if and under which conditions neutrinos do exhibit contextuality indeed. In other words, one asks a natural question concerning physical parameters granting this very quantum property. For this purpose we apply the KCBS test briefly described in the Introduction. It is the simplest, most economic, state-dependent test requiring no more than five observables to be measured. For this purpose one sets the observables A_i in such a way that initial electron neutrino state ($\delta = 0$):

$$|\nu_e\rangle = (c_{12}c_{13}, s_{12}c_{13}, s_{13}) \quad (10)$$

maximally violates the contextuality ($\Delta \approx -3.94$) i.e. it is the centre of the KCBS-pentagon see figure 1. Then the vectors $\{\nu'_1, \nu'_2, \nu'_3, \nu'_4, \nu'_5\}$ creating pentagon around the state $|\nu_e\rangle$ (equation (10)) indicate via the quantity Δ (equation (2)) how the contextuality of the oscillating neutrinos changes in time for a given set of parameters and the different initial states.

In order to obtain the set of vectors $\{\nu'_1, \nu'_2, \nu'_3, \nu'_4, \nu'_5\}$, which maximally violate the KCBS inequality for the given initial electron state it is necessary to rotate all vectors specified in equation (3). First we map the vector $|\psi\rangle = (1, 0, 0)$ into that in equation (3). To do this, it is necessary to find a rotation matrix fulfilling condition: $|\nu_e\rangle = R(\alpha)|\psi\rangle$. Then the transformation of the vectors ν'_i , $i = 1, \dots, 5$ is straightforward: $\nu'_i = R(\alpha)\nu_i$.

In the first step to determine the axis of rotation R one calculates the normalized cross product of the vectors $|\psi\rangle$ and $|\nu_e\rangle$:

$$|\psi\rangle \times |\nu_e\rangle = N(0, -s_{13}, s_{12}c_{13}), \quad N = \frac{1}{\sqrt{s_{13}^2 + s_{12}^2c_{13}^2}}. \quad (11)$$

Afterwards, since:

$$|\psi \times \nu_e| = \sin \alpha, \quad \nu_e \cdot \psi = \cos \alpha, \quad (12)$$

one can easily find an angle α of the rotation $R(\alpha)$.

Then, one can notice that it is plausible to express any rotation as a transformed rotation about z-axis:

$$R(\alpha) = TR_z(\alpha)T^{-1}, \quad (13)$$

where:

$$R_z(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \nu_e \cdot \psi & |\psi \times \nu_e| & 0 \\ -|\psi \times \nu_e| & \nu_e \cdot \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (14)$$

and T is the transformation matrix of the standard basis to the basis consisting of the vectors: $\nu_e, \psi, \psi \times \nu_e$. Since this set is not orthogonal we use the Gram-Schmidt procedure for the vector ν_e :

$$\nu'_e = \frac{\nu_e - (\nu_e \cdot \psi)\psi}{\|\nu_e - (\nu_e \cdot \psi)\psi\|}, \quad \nu'_e = \frac{1}{\sqrt{(s_{12}c_{13})^2 + s_{13}^2}}(0, s_{12}c_{13}, s_{13}). \quad (15)$$

Then one can define the transformation matrix given in equation (13) as:

$$T = (\nu'_e, \psi, \psi \times \nu_e). \quad (16)$$

The simple calculations of equation (13) reveal that:

$$R(\alpha) = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & -s_{13} \\ s_{12}c_{13} & \frac{c_{12}c_{13}^3s_{12}^2 + s_{13}^2}{c_{13}^2s_{12}^2 + s_{13}^2} & -\frac{s_{12}s_{13}c_{13}}{1 + c_{12}c_{13}} \\ s_{13} & -\frac{s_{12}s_{13}c_{13}}{1 + c_{12}c_{13}} & \frac{c_{13}(c_{12} + c_{13})}{1 + c_{12}c_{13}} \end{pmatrix}. \quad (17)$$

Initially, except the 'optimal' electron neutrino ν_e given in equation (10), also ν_μ and ν_τ are natural candidates for initial preparations which, however, to be KCBS-optimal would need to be accompanied by the different measuring operators obtained by an other rotation matrix being, however, analogous to the one given in equation (17). Since the results are in both cases qualitatively the same, we limit our considerations to the neutrinos initially prepared in the electron state and then analyse the KCBS test for the corresponding pentagon.

The quantifier of contextuality $\Delta \leq -3$ is given by the KCBS inequality in equation (2). The time evolution of an initial state equation (10) is obtained via numerical solution of equation (6) which is then imposed into the KCBS test i.e. one calculates Δ for a given time instant. In figure 2 we plot Δ as a function of time t for different values of the coupling V_{CC} indicating the strength of interaction with the normal matter. The set of observables

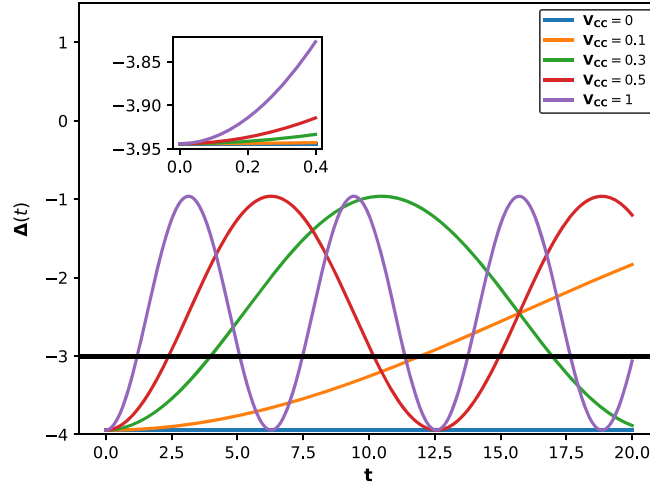


Figure 2. Quantum contextuality parameter Δ as function of time for different values of neutrino–matter coupling V_{CC} is given in meV units. The other parameters are $E = 10$ MeV and $\delta = 0$. The time step in numerical calculations is equal to $\delta t = 0.01$ s. There is an auxiliary line indicating the threshold value for the loss of the contextuality: $\Delta = -3$. *Inset:* a short time behaviour of Δ .

$A_i' = 1 - 2|v_i'\rangle\langle v_i'|$ determined by the requirement of maximal initial violation of the KCBS test obviously remain fixed in time. Initially, see inset in figure 2, the electron neutrino ν_e maximally violates the KCBS inequality as discussed above. Further, the contextuality is satisfied provided that the time-evolving state remains confined in a (relatively) small cone around the state ν_e . The results presented in figure 2 indicate periodicity of Δ which becomes faster with an increase of V_{CC} . In other words, the larger the effect of matter is, the faster neutrino loses its contextuality. On the other hand, an enhancement of V_{CC} results in the quicker return of the neutrino to its initial contextuality and truncation of the time interval during which the neutrino violates the KCBS condition. To sum up, after a rapid contextuality loss indicated by $\Delta > -3$, there occurs its *effective revival* with $\Delta < -3$ solely due to interaction with normal matter i.e. $V_{CC} \neq 0$ followed further by another loss. We use here the adjective ‘effective’ since the effect is transient and for sufficiently long time there is no qualitative difference between the systems with the different values of V_{CC} . The value of V_{CC} describing the size of a matter effect is chosen very large $V_{CC} \sim 1$ meV. Although such a large value of the V_{CC} coupling can in principle be achieved in astrophysics e.g. in neutron stars [36] or supernovae with a matter density of order 10^{11} g cm $^{-3}$ [39, 40], we emphasize an essentially model character of our calculations. In particular the range of the parameter t is chosen suitable to express the qualitative rather than quantitative effects since it is comparable (or even larger) with a duration of coherence loss of neutrino in a Earth matter. Further increasing V_{CC} would decrease the time needed for the first contextuality revival. In other words, the effect described here requires very restrictive and tailored conditions to occur.

4. Decoherence effect: neutrino as an open system

The neutrino oscillations are mostly treated in the framework of the closed quantum systems and described according to the above mentioned unitary evolution. Nevertheless, every physical system—in particular that which is coupled to a matter, as it is the case of neutrinos—is subjected, at least to small extent, to decoherence effects originating from interaction with the environment and resulting in *non-unitary* corrections to its time evolution [41]. The system, treated as a whole, is assumed to be closed and evolves unitarily, while the *reduced* evolution of the (open) subsystem is determined by a non-unitary operator $\rho(t_j) = \Gamma(t_j, t_i)\rho(t_i)$ where $\rho(t_i)$ is the reduced density matrix of the time-evolving system [42]. Obviously the operator Γ cannot be arbitrary—the transformation must be completely positive [42–44] and satisfying the semi-group property i.e. obeying the composition law: $\Gamma(t_2, t_0) = \Gamma(t_2, t_1)\Gamma(t_1, t_0)$. In particular, for $\Gamma(t_j, t_i) = \Gamma(t_j - t_i)$ the time evolution is Markovian. Such conditions are fulfilled by the neutrino’s density matrix $\rho_F(t)$ obeying the Lindblad–Kossakowski Markovian master equation ($\hbar = 1$) [41, 45]:

$$\frac{d}{dt}\rho_F = -i[\mathcal{H}_F, \rho_F] + L[\rho_F], \quad (18)$$

where initially we assume neutrino in a particular flavour state $\rho_F(0) = |\Psi_F\rangle\langle\Psi_F|$ which in our case is chosen to be electron i.e. $|\Psi_F\rangle = |\nu_e\rangle$. The Hamiltonian part of equation (18) is generated by \mathcal{H}_F defined in equation (7). The (non-Hamiltonian) Lindbladian part of the master equation equation (18):

$$L[\rho_F] = \sum_{i,j=0}^{N^2-1} c_{ij} (F_i \rho_F F_j^\dagger - \frac{1}{2} \{F_j^\dagger F_i, \rho_F\}) \quad (19)$$

is responsible for the non-standard effects connected with dissipation and decoherence. Here $N = 3$ denotes the dimension of the system, $\{\cdot, \cdot\}$ is an anti-commutator, the matrices F_n stand for the generators of $SU(N)$ and c_{ij} are coefficients satisfying the set of following inequalities (in order to have all the properties, including complete positivity, of the map Γ retained [44, 46]):

$$|c_{ij}| \leq \frac{1}{2}(c_{ii} + c_{jj}). \quad (20)$$

Moreover, as the decoherence effects has been proposed as a possible explanation of certain experimental data [47–50], also the constraints on decoherence parameters are related to experimental data [51, 52]—see [34] for a summary of a recent progress on that topic. Since in our case the dimension $N = 3$, the matrices F_n are generators of $SU(3)$ and in the standard representation are given by the celebrated Gell–Mann matrices $F_n = \frac{\lambda_n}{2}$, where λ_n [53]:

$$\begin{aligned} \lambda_0 &= \mathbb{I}_3, \quad \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned} \quad (21)$$

Let us emphasize that equation (18) is written in the flavour basis. The other option is to use the mass-states basis instead. However, the choice adapted and privileged here is related to a basis of *measurable* quantity—the flavour i.e. it is related to the *pointer states* [54] of the system and, as that, natural for open systems. Let us also notice that the Kossakowski–Lindblad form is invariant under changes of a basis but an interpretation of coefficients becomes different. Obviously, if one applies, instead of phenomenological, rigorous approach, one could—at least in principle—find a physically rather than mathematically (or formally) justified form of time evolution of the system.

Let us notice that equation (18) can be, following [44], rearranged to the form which, not including any additional physical content, is of particular usefulness in numerical calculations:

$$\frac{d}{dt}[\rho_F]_v = \mathcal{H}_{\text{eff}}[\rho_F]_v, \quad \mathcal{H}_{\text{eff}} = \mathcal{H} + \mathcal{D}, \quad (22)$$

which allows to extract the ‘Hamiltonian’ \mathcal{H} and ‘dissipative’ \mathcal{D} (i.e. present in the open systems only) part of an effective time evolution generator \mathcal{H}_{eff} similarly as it was done in [30]. The notation $[\rho_F]_v$ means that the density matrix is now represented by a 9-dimensional coherence-vector:

$$[\rho_F]_v = (\rho_0, \rho_1, \dots, \rho_8), \quad (23)$$

where coefficients ρ_i , $i = 0, \dots, 8$ are such that:

$$\rho_F = \sum_{i=0}^8 \rho_i F_i. \quad (24)$$

The matrices \mathcal{H} and \mathcal{D} represent the ‘ordinary’ and dissipative (decohering) part of the Hamiltonian, respectively. Their explicit form is however too complicated (they are 9-dimensional matrices) to be reproduced here. At www.dropbox.com/sh/m678lxyn6fek2h6/AABM1Kg7wczX0JsRU-j2gcsHa?dl=0 one can find Mathematica code for their generation. In particular, following [44], after transforming the original, native problem equation (18) into the coherence-vector form equation (22) the numerical tool which is utilized is a Python-based exponentiation of finite dimensional matrices.

As one can expect, provided that it is sufficiently large, an effect of decoherence is lethal for contextuality as plotted in figure 3 for the simplest case of the constant Lindblad–Kossakowski matrix $c_{ij} \equiv c$, $\forall i, j$. The larger the amplitude of c_{ij} is, the faster the contextuality parameter Δ crosses the line $\Delta = -3$ indicating contextuality loss. However, even in the presence of decoherence the contextuality revivals of contextuality are possible provided that the neutrinos are coupled to the normal matter with a sufficiently large V_{CC} . This behaviour is

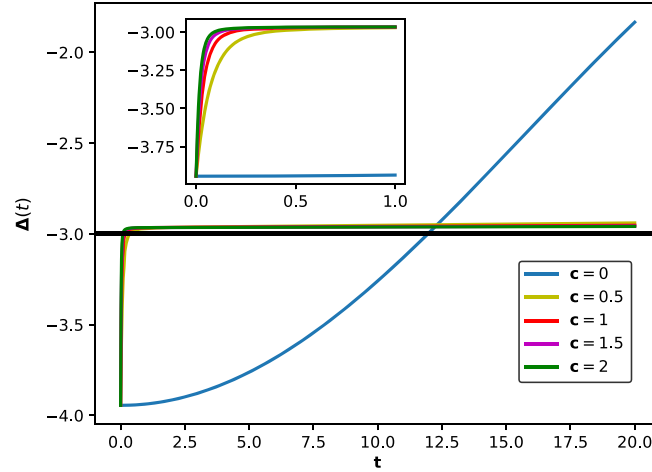


Figure 3. Quantum contextuality parameter Δ as function of time for different values of the elements of the Lindblad–Kossakowski matrix $c_{ij} \equiv c$ given in meV and fixed $V_{CC} = 0$. The other parameters are $E = 10$ MeV and $\delta = 0$. The time step in numerical calculations is equal to $\delta t = 0.01$ s. There is an auxiliary line indicating the threshold value for the loss of the contextuality: $\Delta = -3$. *Inset:* a short time behaviour of Δ .

depicted in figure 4. Initially one observes fast contextuality loss indicated by $\Delta > -3$ but then, e.g. for $V_{CC} = 0.3$ meV due to its oscillatory behaviour again $\Delta < -3$ i.e. neutrino regains its contextuality.

Both the decoherence and the neutrino–matter interaction are responsible for the above mentioned revivals of contextuality. One could expect the first of these two mechanisms to be rather destructive. It is clearly the case but the way how the decoherence affects contextuality of neutrino is highly related to V_{CC} . The purity $\mathcal{P} = \text{Tr}\rho^2$ [41] is a natural quantifier for a ‘decoherence strength’: it is maximal for pure states which are projectors satisfying the idempotence condition $\rho^2 = \rho$ whereas minimal for maximally mixed states ρ proportional to an identity matrix. There is a relation between contextuality revivals of contextuality and the purity $\mathcal{P} = \text{Tr}\rho^2(t)$ of the time-evolving neutrino. In the inset of the upper panel of figure 4 it is presented that the purity, due to environmental coupling, decreases (on average) with time but, for certain values of V_{CC} oscillates. There is a relation between (decaying) oscillations of both the contextuality and the purity indicating a relation between decoherence and an interaction with matter leading to optimal parameters for transient revivals of contextuality. Let us emphasize, however, that in a presence of strong decoherence, as presented in the lower panel of figure 4 contextuality revivals are absent since the system purity loss is very rapid.

Let us indicate an apparent similarity between the contextuality decay and revivals of contextuality reported here and the entanglement sudden death [55] and rebirth [56]. This well known and celebrated effects peculiar for time-evolving open quantum systems [57] exemplify another analogy between different notions of quantum non-locality [4, 5].

5. Summary

Neutrino oscillations are one of most fundamental realization of the quantum three-level systems [35] exhibiting all the quantum ‘unusual’ features with the contextuality as a particular example. In our work, we investigated the relation between contextuality quantified by the KCBS 5-observable test and the effect of matter interacting with neutrinos and showed that this interaction results in effective revivals of contextuality in time. This counter-intuitive property is of a transient character and hence is related to a time period indicating duration of evolution. To be precise: one cannot *globally* enhance contextuality solely due to matter interaction but, as we showed, one can expect contextuality loss and revivals of contextuality occurring more frequently in a certain time interval in a presence of matter affecting neutrino’s oscillation in comparison to the case where the matter is absent. Effective revivals of contextuality remain also present in neutrinos’ oscillation affected by Markovian decoherence phenomenologically modelled via Kossakowski–Lindblad master equation equation (18). The Kossakowski–Lindblad master equations describing decoherence (with complete positivity as an underlying guideline for their construction) are fairly general despite of their Markovian limitation. They allow for credible predictions of time-evolving quantum systems weakly coupled to environment. Clearly, for large amplitudes of Kossakowski–Lindblad dissipations in equation (18) contextuality is lost as the system became effectively classical. Nevertheless, we showed that for weak and intermediate decoherence revivals of contextuality remain present. We also indicated that there is a relation between contextuality revivals and the

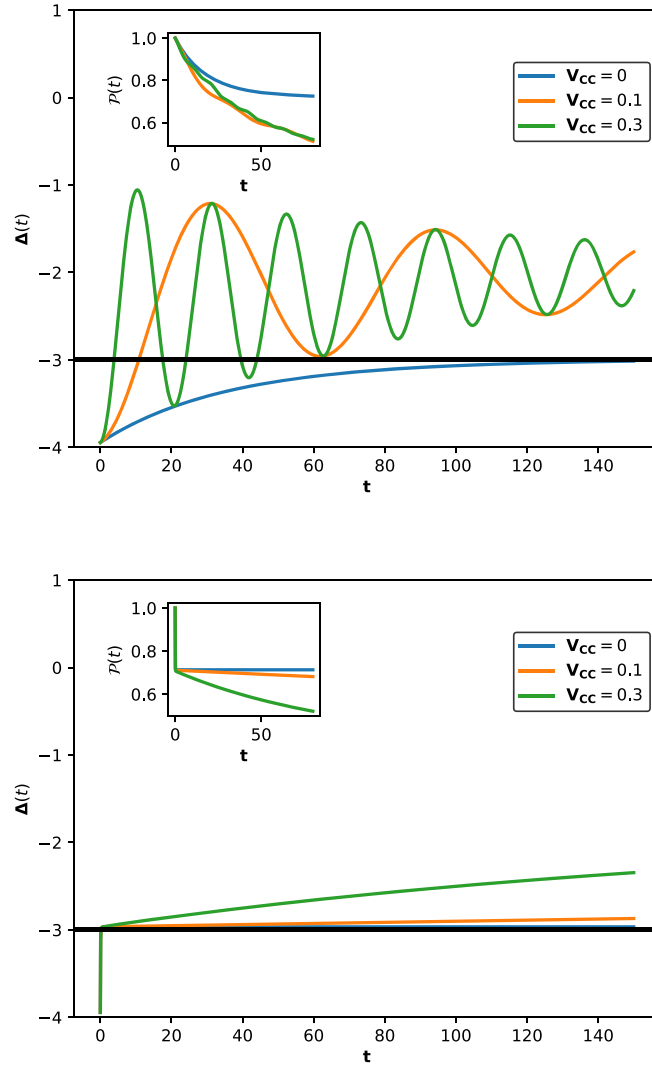


Figure 4. Quantum contextuality parameter Δ as function of time for different values of the neutrino–matter coupling V_{CC} (given in meV) and the elements of the Lindblad–Kossakowski matrix $c_{ij} = 0.001$ meV (*upper panel*) and $c_{ij} = 1$ meV (*lower panel*). There is an auxiliary line indicating the threshold value for the loss of the contextuality: $\Delta = -3$. For $V_{CC} = 0$ in an upper panel contextuality loss $\Delta(t) > -3$ occurs for larger values of time than indicated on the plot. *Insets:* purity $\mathcal{P} = \text{Tr}\rho^2$ of a neutrino’s system evolving according to equation (18) as function of time for different values V_{CC} given in meV. The other parameters are $E = 10$ MeV and $\delta = 0$. The time step for numerical calculations $\delta t = 0.1$ s.

purity of the system calculated for the neutrino state solving equation (18). For large decoherence, when the purity is monotonically lost one cannot expect contextuality revivals which do occur together with a (damped) oscillations of purity indicating time intervals of faster and slower purity loss.

Three-dimensional systems—qutrits—are the simplest objects exhibiting contextuality yet rich enough to attract attention [26, 58]. A ‘natural qutrits’—neutrinos—exhibiting often unexpected properties and attracting researchers from very different sub-disciplines of physics are a stage of the most fundamental properties of a quantum world. Contextuality, one of such properties, is one more test for our ability of acceptance counter-intuitivity of Nature. Our studies are based on two main model assumptions: (i) the first, concerning neutrinos, is formalized in equation (6); (ii) the second is about the utilized decoherence approximation given in equation (18) with the Kossakowski–Lindblad underlying guidelines and the complete positivity of time evolution among them. We hope that our theoretical investigations performed from a quantum mechanical perspective can contribute, yet modestly, to a better understanding quantum properties of neutrinos.

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